Two-fraction model of two-dimensional electron systems on thin helium films investigated with cyclotron resonance

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Introduction & Motivation

Cyclotron resonance (CR) measurements of two-dimensional electron systems (2DES) on thin helium films show an increasing asymmetry of the CR-line when the helium film thickness is reduced. So far there was no satisfying explanation of this effect. We are presenting a new theoretical approach taking the roughness of the substrate-surface into account. On a rough surface **two fractions** of electrons occur, free electrons and electrons localized to tops of the surface roughness. The well known CR-line for free electrons is then modified by the contribution of the localized electrons, which gives rise to the asymmetry.

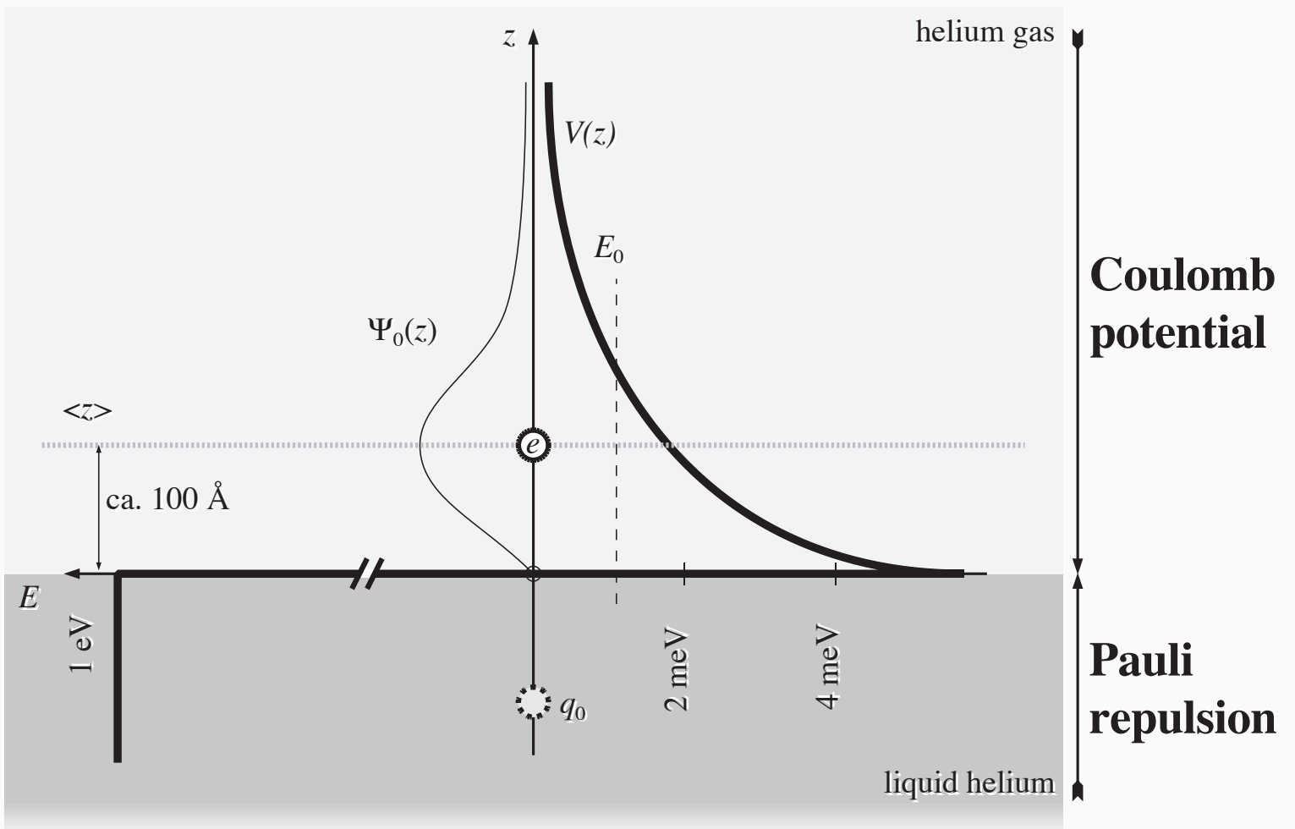
The experimental results demonstrate the application of the model.

Electrons on liquid Helium

Electrons on liquid helium feel a hydrogen-like potential

$$V(z) = \begin{cases} V_0 & z \le 0 \\ -\frac{1}{4\pi\epsilon_0}\frac{q_0 e^2}{z+\beta} & z > 0 \end{cases} \quad \text{with:} \quad \begin{aligned} V_0 &\approx 1\text{eV} \\ q_0 &= \frac{\epsilon_{\text{L4He}}-1}{4(\epsilon_{\text{L4He}}+1)} \end{aligned}$$

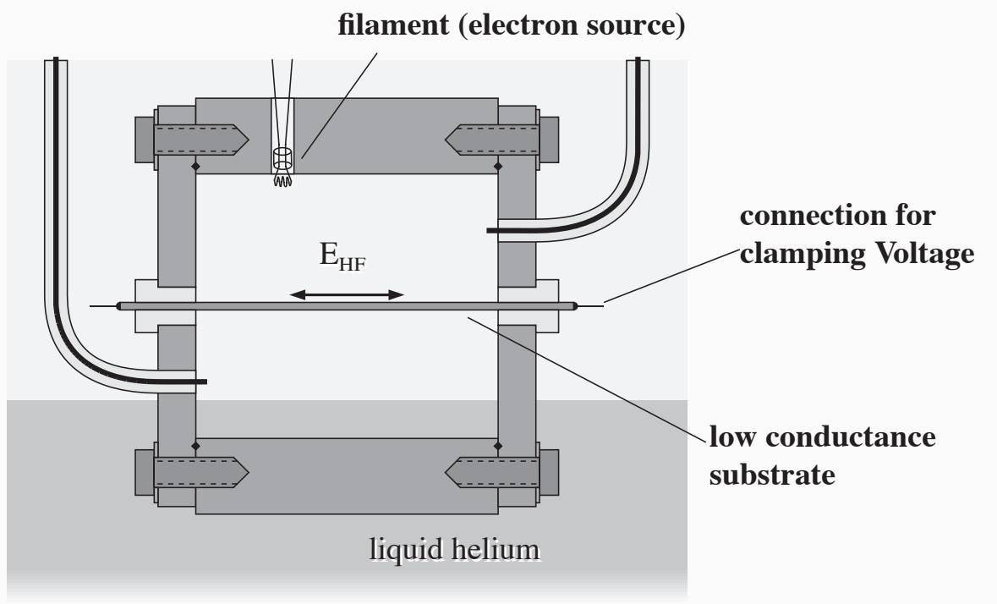
If one assumes V_0 to be infinite, the resulting wavefunctions are similar to the well-known wavefunction of the hydrogen atom.

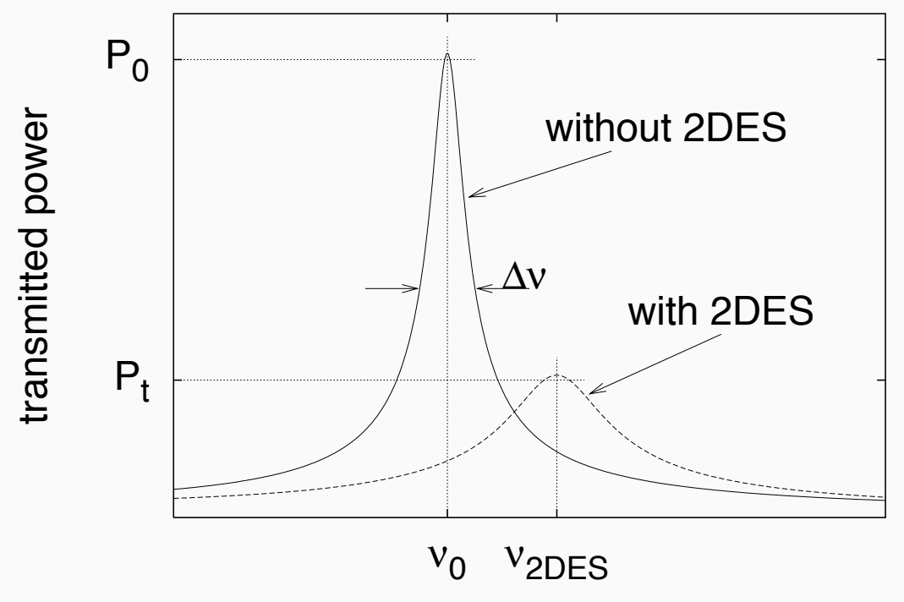


In the experiment, *T* is around 1.3K. At this temperature most of the electrons are in the ground state and therefore form a real 2D layer on top of the liquid helium.

Experimental Setup

A schematic drawing of the used microwave cavity is shown. It is driven in the fundamental TM010 mode, where the maximal amplitude of the electrical field is in the vicinity and parallel to the substrate.





The dependence of the resonance of the cavity with and without a 2DES on the helium film on the substrate is shown.

The two-fraction Model

Modelling the Helium film thickness

For structures much smaller than the capillary length of liquid helium one can extract the properties of a neutral helium film *d(x)* from

$$\sigma_{lv}\frac{d''(x)}{[\,1+(d')^2\,]^{3/2}} - \rho g d(x) + \frac{C_3}{d^3(x)} = \rho g h$$

where C_3 is the van-der-Waals constant of the helium-substrate boundary. The radius of curvature of the capillary condensed film is defined as

$$\frac{2\sigma_{lv}}{R} \simeq \rho g h$$

Modelling the substrate roughness

We assume that a 1-dimensional roughness behaviour $\delta(x)$ can be described by a Gaussian distribution of the amplitudes

$$G(\delta) = \frac{1}{\sqrt{2\pi\,\Delta^2}}\exp\left[-\frac{\delta^2}{2\Delta^2}\right]$$

Here only the high enough tops of the substrate roughness above a the fixed level $\delta > 0$ play a role. Their density is given by

$$n_\delta = \frac{1}{\sqrt{2\pi}\,\eta}\exp\left(-\frac{\delta^2}{2\Delta^2}\right) \quad \text{with} \quad \eta = \sqrt{\langle \eta^2 \rangle}$$

This gives, with some limitation, the density of the active tops n_a^T.

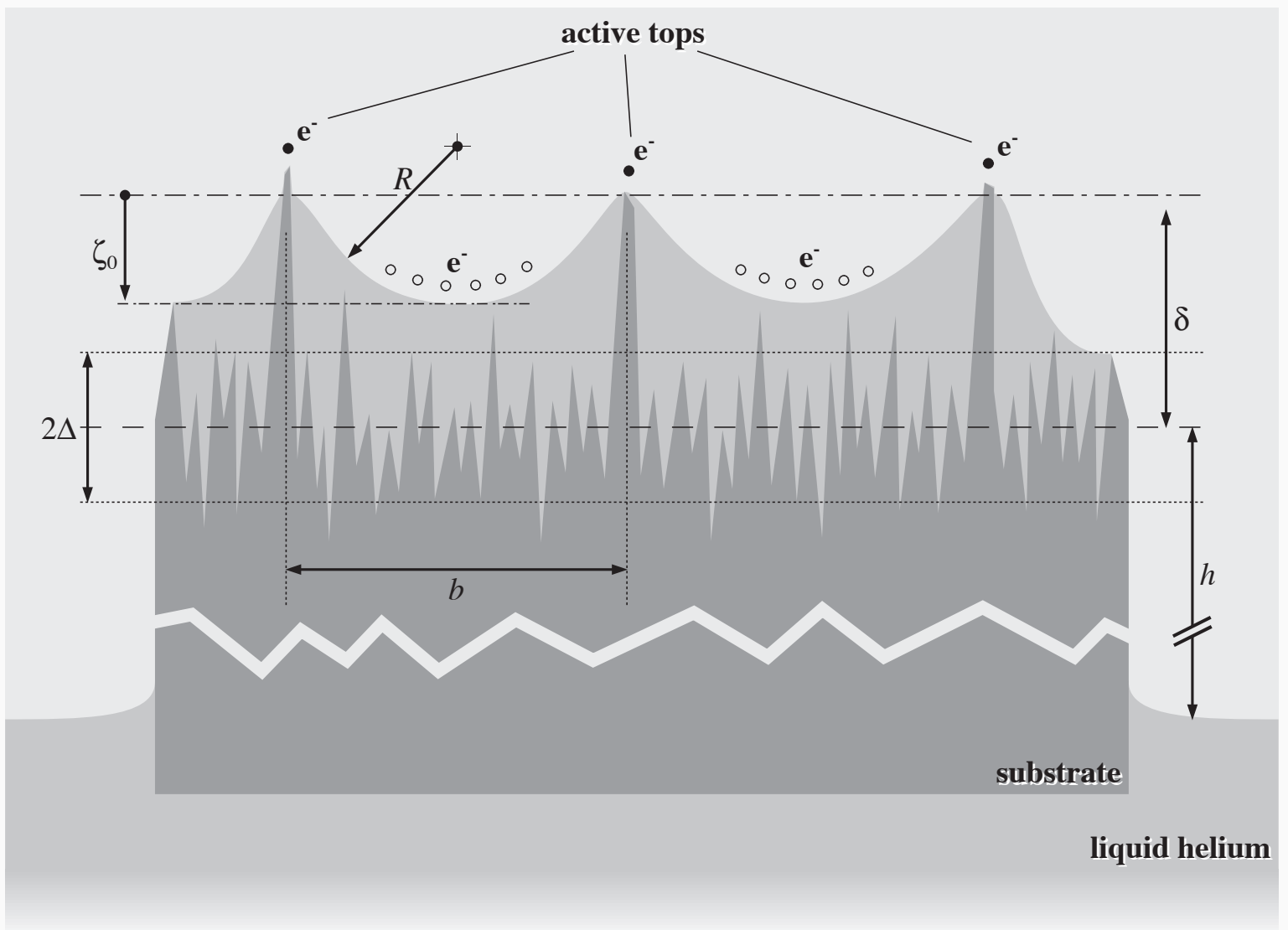
Properties of the two fractions

The total electron density n_s is the sum of free and localized density

$$n_e + n_l = n_s$$

The relationship between these fractions is flexible and is determined by the chemical potential ${}_0$. The definition of ${}_0$ is taken from semiconductor physics.

$$n_l = \frac{n_a}{\exp[(V_a - \mu_o)/T] + 1}\,, \quad V_a < 0 \qquad n_e = \frac{n_o^e \exp(T_e/T)}{\exp(-\mu_o/T) + 1}\,, \quad n_o^e = \frac{mT}{(2\pi\hbar^2)}$$



Results for CR in the microwave cavity

The absorption of the free electron fraction is

$$Q_e^{-1} \;\propto\; n_e\frac{1+z+x}{(1-z+x)^2+4z}$$

The absorption of the localized electron fraction n_l is

$$Q_l^{-1} \;\propto\; n_l\frac{\arctan\frac{\sqrt{z}}{1+x+\sqrt{xz}} + \arctan\frac{\sqrt{z}}{(1+x)\sqrt{z}-z\sqrt{x}} + c(z,x)}{2\sqrt{z}}$$

Here $z = \omega_o^2\,\tau^2$ and $x = \omega_c^2\,\tau^2$. So the total absorption of the 2DES in an external magnetic field is given by the sum of its parts:

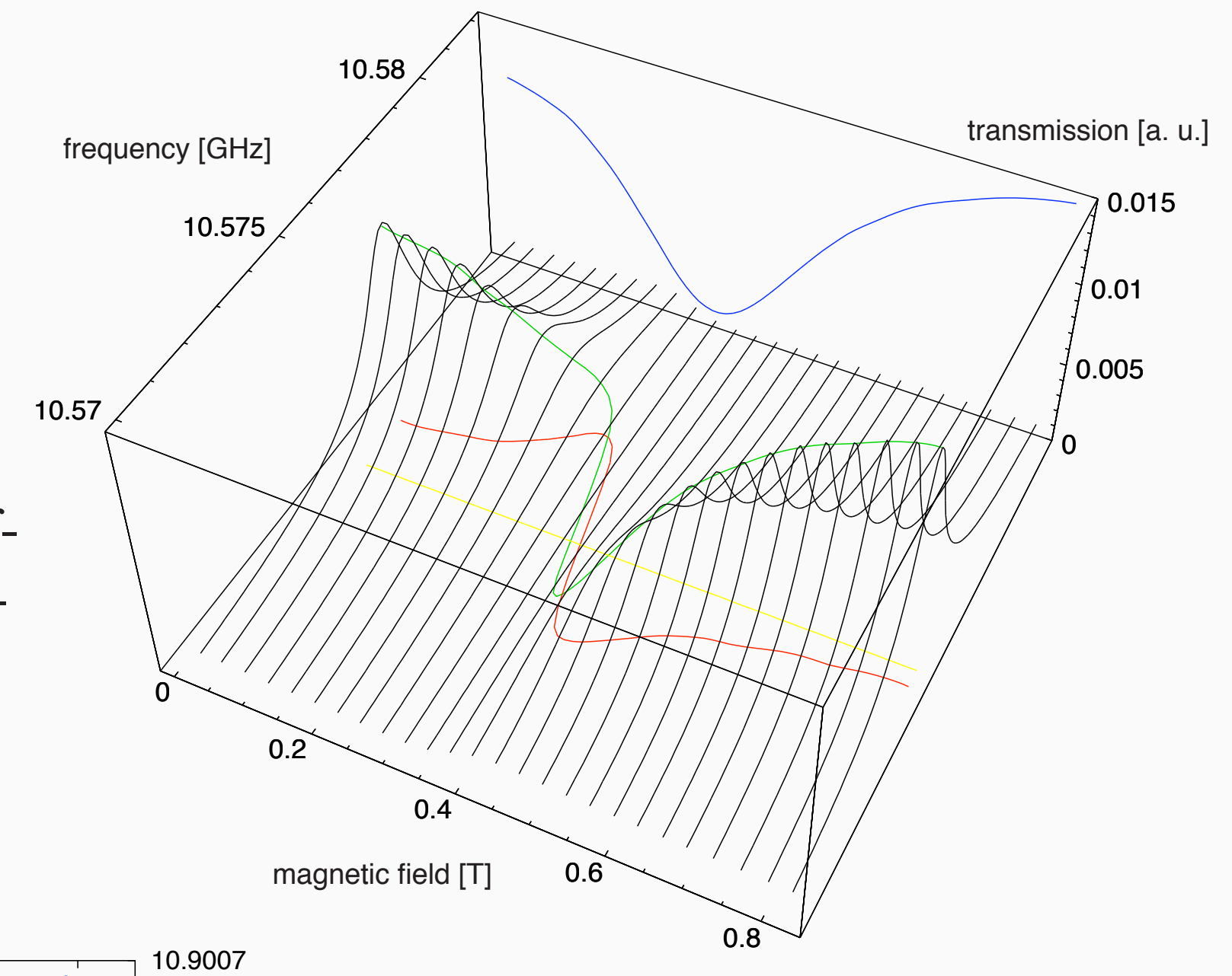
$$Q^{-1} = Q_e^{-1} + Q_l^{-1}$$

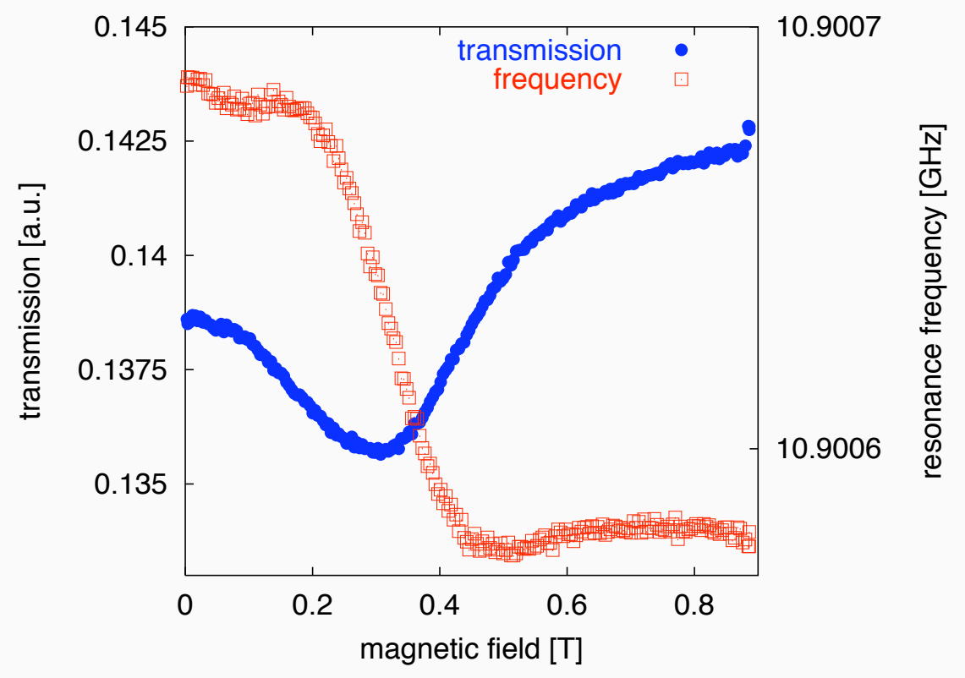
Experiments

How the experiment works

After charging the helium film on the substrate with a 2DES, the magnetic field is swept from 0 to around 0.9 T.

The resonance of the cavity is recorded for different values of the magnetic field. (Here for a 2DES on bulk He).





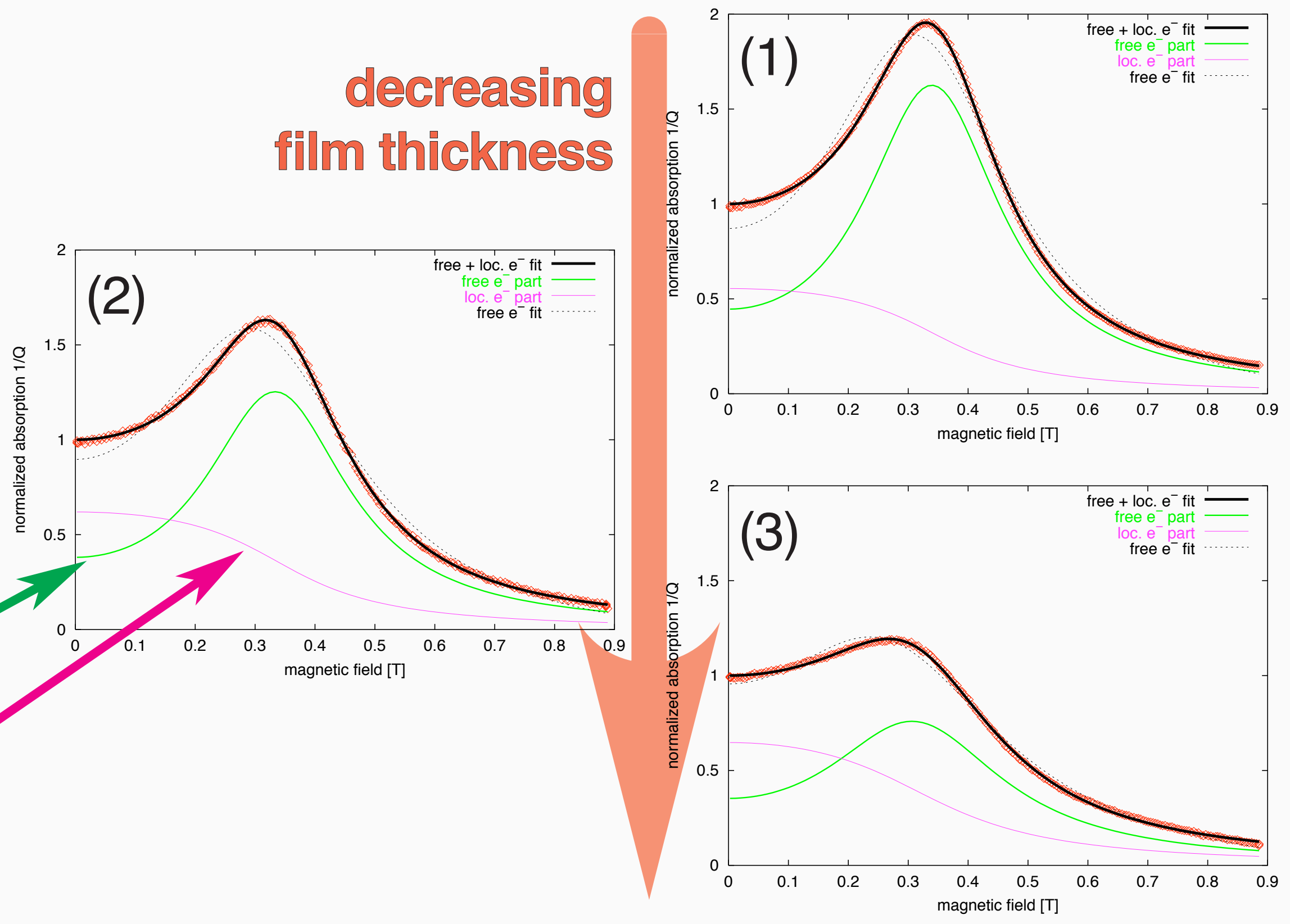
The position of the resonance maximum in transmission and frequency gives the typical CR data.

Fitting the results

The reciprocal of the experimental transmission data is fitted to Q^{-1}, with the following 5 free parameters:

$$\omega,\ \tau,\ A_{\text{free electrons}},\ A_{\text{localized electrons}}\ \text{ and } c_{\text{constant offset}}$$

Here the fit results, normalized to $Q^{-1}(B = 0\text{ T})$, are shown. The green and the pink lines represent the free and localized electron fraction, the **full** line is the sum of both, fitted to the data:



To obtain the values of the

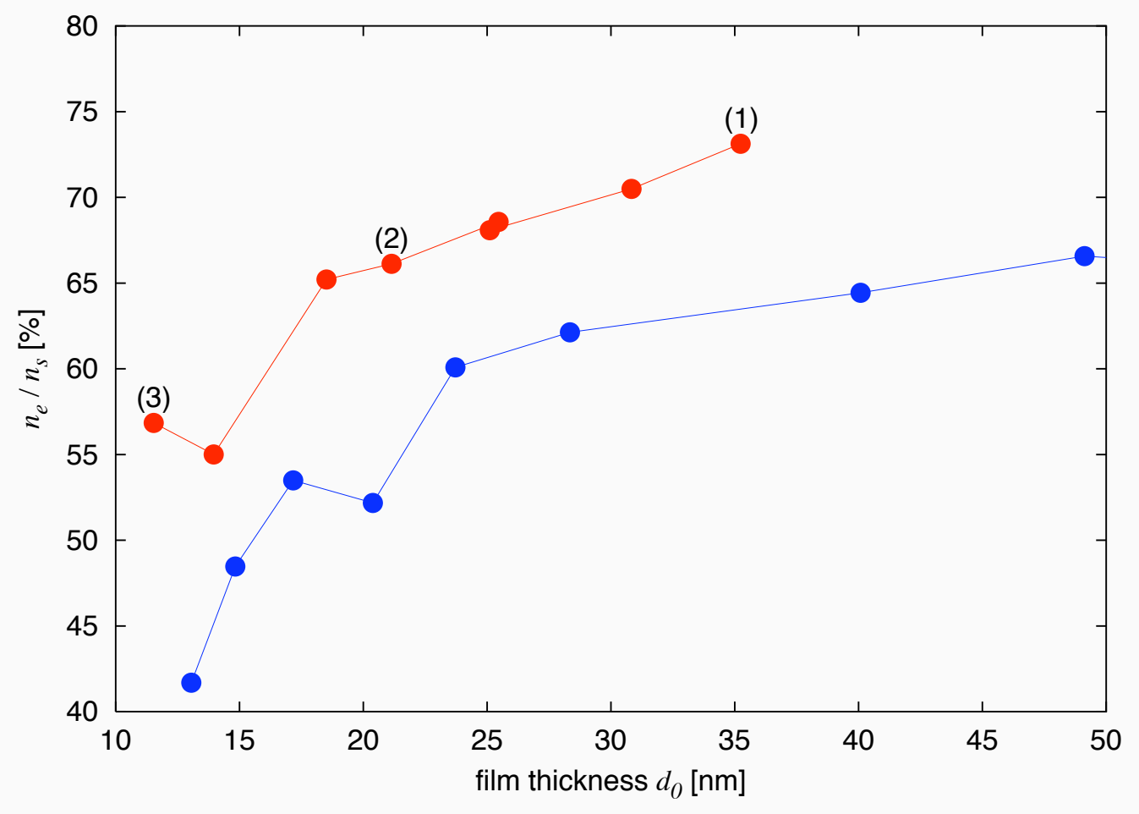
$$\text{free } \nu_e = \frac{n_e}{n_s} \text{ and localized } \nu_l = \frac{n_l}{n_s} \text{ electron fraction,}$$

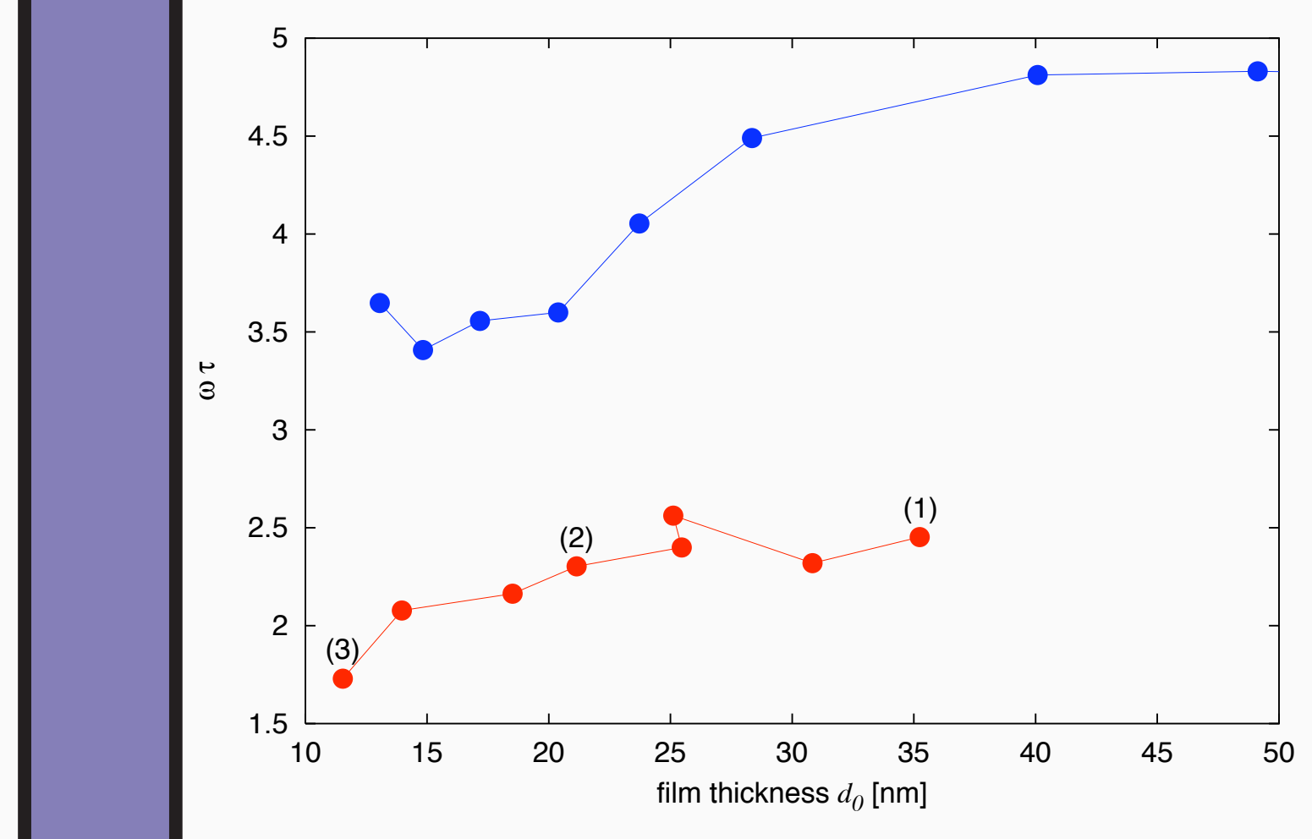
the ratio between the maximum and the zero field value of Q^{-1} is used:

$$\frac{Q^{-1}(\omega_c^{(max)})}{Q^{-1}(\omega_c = 0)} = \frac{\nu_e\; p(\omega_o, \tau, \omega_c^{(max)}) + \nu_l\; q(\omega_o, \tau, \omega_c^{(max)})}{\nu_e\; p(\omega_o, \tau, 0) + \nu_l\; q(\omega_o, \tau, 0)}$$

Analyzing the data

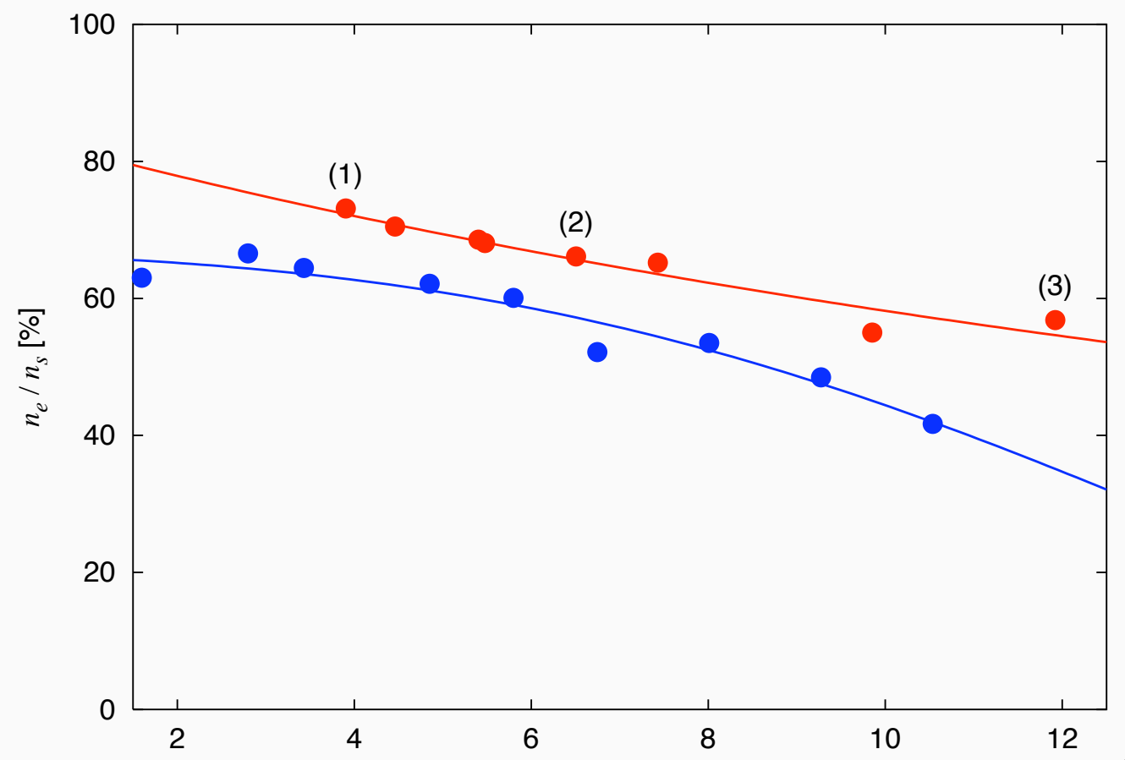
The free electron fraction vs thickness of the helium film shows qualitatively the expected behaviour. The red data-set is measured on a very smooth SiO_2 on Silicon substrate. The blue data-set is taken on PMMA on Silicon substrate (by G. Mistura). The dip in the thin film region may be related to the so-called dip-effect, described by *Shikin et al., PRB* ***64*** *(2001)*





The behaviour of ωτ vs the thickness of the helium film is, at least qualitatively as expected. Unfortunately the presented theory is too crude to provide a detailed interpretation.

Fit of the model parameters of the localized electrons to the data. From the data on PMMA we get Δ around 8nm and η around 6nm. These values, being in the nm-regime, are reasonable and typical for the surfaces used in the experiments.



Conclusions

The two-fraction model proves once more to be suitable for explaining the behaviour of electrons on helium films and has the following benefits:

- The origin of CR-line asymmetry of a 2DES on thin helium films is understandable, and a quantitative analysis of the electron fraction is possible.
- The method is a good candidate for the characterization of the substrate surface. This is very useful for measurements at high electron densities.

Outlook

Experimentally:

- Perform film-thickness dependend CR measurement for a wider range of film thickness and substrate roughness.
- Do precise measurements to resolve the dip and its origin in the n_e/n_s vs d_0 diagram for very thin films.

Theoretically:

- Refine theory to be suitable for quantitative data analysis.
- Consider special properties of the cavity.

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